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MODERN METHODS OF MULTIPLE SPECTRAL DENSITY ESTIMATION.(U)

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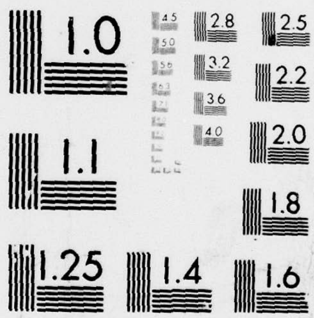

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INSTITUTE OF STATISTICS

MODERN METHODS OF MULTIPLE SPECTRAL

DENSITY ESTIMATION

by H. Joseph Newton

Texas A&M University

Technical Report No. N-8

March 1979

Texas A & M Research Foundation
Project No. 3838

"Multiple Time Series Modeling and Time
Series Theoretic Statistical Methods"

Sponsored by the Office of Naval Research

Professor Emanuel Parzen, Principal Investigator

Approved for public release; distribution unlimited.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: Technical Report No. N-8 (14) TR-8	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): (6) Modern Methods of Multiple Spectral Density Estimation		5. TYPE OF REPORT & PERIOD COVERED: (9) Technical rept.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): (10) H. Joseph Newton		8. CONTRACT OR GRANT NUMBER(s): (15) N00014 78-C-0599
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Texas A&M University, Institute of Statistics, College Station, TX 77843		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Code 436, Arlington, VA. 22217		12. REPORT DATE: (11) March 1979
		13. NUMBER OF PAGES: 31
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): (12) 17		15. SECURITY CLASS. (of this report): Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report): NA		
18. SUPPLEMENTARY NOTES: NA		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number): Multiple Time Series, Spectral Density Function, Autoregressive Process, Periodic Autoregressive Process.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number): Methodology for estimating the spectral density of a multiple time series is presented. The windowed and autoregressive methods are described and a new estimator called the periodic autoregressive estimator is proposed. A worked example is given and a Fortran program written by the author is described.		

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1. Introduction and Summary

A statistical tool of increased use in many scientific areas is the spectral density function of a d-dimensional time series. Let Z be the set of integers and $\{\underline{Y}(t),\ t\in Z\}$ be a d-dimensional, Gaussian, zero mean, covariance stationary time series with autocovariance function $\{R(v),\ v\in Z\}$, i.e., $\underline{Y}(t)$ is a d-vector with j^{th} component $Y_j(t)$ and $R(v)$ is a $d\times d$ matrix with typical element $R_{jk}(v) = E(Y_j(t)Y_k(t+v))$. Thus $R(v) = E(\underline{Y}(t)\underline{Y}^T(t+v))$ where A^T denotes the transpose of the matrix A.

The joint distribution of a finite stretch $\underline{Y}(1), \ldots, \underline{Y}(T)$ of $\underline{Y}$ is thus determined by $R(\cdot)$. An equivalent parametrization of $\underline{Y}$ is its spectral density function. If for all (j, k), $\sum_{V=-\infty}^{\infty} |R_{jk}(v)| < \infty$, then there exists a $d\times d$ Hermitian complex matrix function $f(\omega)$, $\omega\in[-\pi, \pi]$, called the spectral density function, related to $R(\cdot)$ by

$$f(\omega) = \frac{1}{2\pi}\sum_{V=-\infty}^{\infty} R(v)e^{-iv\omega}, \qquad \omega\in[-\pi, \pi]$$

$$R(v) = \int_{-\pi}^{\pi} f(\omega)e^{iv\omega}\, d\omega, \qquad v\in Z .$$

The spectral density is extremely useful in describing individual component series $\{Y_j(t),\ t\in Z\}$ and in determining relations among several such series.

We can write the Cramér represe tation of $\underline{Y}$ (see [2], p. 104 for example):

$$\underline{Y}(t) = \int_{-\pi}^{\pi}[\cos \omega t\ d\underline{U}_Y(\omega) + \text{Sin}\ \omega t\ d\underline{V}_Y(\omega)]$$

$$\equiv \underset{N\to\infty}{\text{L.I.M.}}\ \frac{2\pi}{N}\sum_{n=0}^{N-1}\{\cos\ \omega_n t\ [\underline{U}_Y(\omega_{n+1}) - \underline{U}_Y(\omega_n)]$$

$$+ \text{Sin}\ \omega_n t[\underline{V}_Y(\omega_{n+1}) - \underline{V}_Y(\omega_n)]\}$$

where $\omega_j = 2\pi j/N$, L.I.M. denotes limit in mean square, and $\underline{U}_Y(\cdot)$, $\underline{V}_Y(\cdot)$ are d-dimensional continuous parameter stochastic processes with independent increments (see [10], p. 26). Further, in differential *notation*

$$\text{Cov}(d\underline{U}_Y(\omega_j),\ d\underline{U}_Y(\omega_k)) = \frac{1}{2}(\delta_{\omega_j-\omega_k} + \delta_{\omega_j+\omega_k})\text{Re } f(\omega_j)d\omega_j d\omega_k$$

$$\text{Cov}(d\underline{U}_Y(\omega_j),\ d\underline{V}_Y(\omega_k)) = \frac{1}{2}(\delta_{\omega_j-\omega_k} - \delta_{\omega_j+\omega_k})\text{Im } f(\omega_j)d\omega_j d\omega_k \qquad (1)$$

$$\text{Cov}(d\underline{V}_Y(\omega_j),\ d\underline{V}_Y(\omega_k)) = \frac{1}{2}(\delta_{\omega_j-\omega_k} - \delta_{\omega_j+\omega_k})\text{Re } f(\omega_j)d\omega_j d\omega_k ,$$

where Re $f(\omega_j)$ and Im $f(\omega_j)$ denote the real and imaginary part of the matrix $f(\omega_j)$, and $\delta_c = 1$ if $c = 0$ and $\delta_c = 0$ otherwise.

This representation allows one to approximately decompose the time series $\underline{Y}$ into the sum of frequency components

$$\underline{Y}(t) \doteq 2\sum_{n=0}^{N-1}[\cos\ \omega_n t\ d\underline{U}_Y(\omega_n) + \text{Sin}\ \omega_n t\ d\underline{V}_Y(\omega_n)]$$

and the variations in $\underline{Y}$ due to the components in a given frequency range are assessed by (1).

Thus

$$\mathrm{Var}(Y_j(t)) = R_{jj}(0) = \int_{-\pi}^{\pi} f_{jj}(\omega)d\omega \doteq \frac{2\pi}{N}\sum_{n=0}^{N-1} f_{jj}(\omega_n)$$

$$\mathrm{Cov}(Y_j(t), Y_k(t)) = R_{jk}(0) = \int_{-\pi}^{\pi} f_{jk}(\omega)d\omega \doteq \frac{2\pi}{N}\sum_{n=0}^{N-1} f_{jk}(\omega_n)$$

and $f_{jj}(\omega)$ is called the power spectrum for series $Y_j(\cdot)$, while $W_{jk}(\omega) = |f_{jk}(\omega)|/(f_{jj}(\omega)f_{kk}(\omega))^{\frac{1}{2}}$ is called the coherency spectrum of series $Y_j(\cdot)$, $Y_k(\cdot)$.

Another important use of multiple spectra is in building and describing linear filters used in describing possibly time lagged relationships among the univariate series. Further the effects of such filters on frequency components can be studied via the spectral density.

Thus, if $\hat{\underline{Y}}(t)$ is the output of a filter with input $\underline{Y}(t)$, i.e. for $r \times d$ matrices b_s,

$$\hat{\underline{Y}}(t) = \sum_{s=-\infty}^{\infty} b_s \underline{Y}(t - s) ,$$

then the spectral density of the input and output are related by

$$f_{\hat{Y}}(\omega) = B(\omega)f_Y(\omega)B^*(\omega)$$

where $B(\omega) = \sum_{s=-\infty}^{\infty} b_s e^{-is\omega}$, and A^* denotes the complex conjugate transpose of the matrix A. The frequency transfer function $B(\cdot)$ can thus be used to determine the b and describe the affect on frequency components of the filter in various frequency ranges. See [11] for a description of various functions derived from $B(\cdot)$ that are useful in this regard.

Section 2 of this paper describes methods in general use for estimating $f(\cdot)$, Section 3 considers the autoregressive spectral estimator, and in Section 4 a new estimator is proposed based on the concept of a periodically stationary autoregressive process ([3], [7]). In Section 5

a worked example is presented which uses a computer program written by the author. The program itself is described in Section 6.

2. Traditional Methods

Given a sample realization $\underline{Y}(1), \ldots, \underline{Y}(T)$ from $\underline{Y}$, a natural estimator of $f(\omega)$ can be obtained by estimating $R(v)$, $|v| < T$, by the positive definite sample autocovariances

$$R_T(v) = \frac{1}{T}\sum_{t=1}^{T-v} \underline{Y}(t)\underline{Y}(t + v) = R_T^{\,T}(-v) , \qquad v = 0, \ldots, T - 1 ,$$

and forming the sample spectral density (or periodogram)

$$f_T(\omega) = \frac{1}{2\pi}\sum_{|v|<T} R_T(v)e^{-iv\omega} , \qquad \omega\epsilon[-\pi, \pi]$$

$$= \frac{1}{2\pi T} W(\omega)W^*(\omega) ,$$

where

$$W(\omega) = \sum_{t=1}^{T} \underline{Y}(t)e^{it\omega} .$$

This estimator suffers from a lack of consistency as shown by

<u>Theorem</u> ([4], p. 249)

Let A be an $n \times n$ matrix having columns $\underline{a}_1, \ldots, \underline{a}_n$. Define the $n^2 \times 1$ vector vec(A) as $\mathrm{vec}(A) = (\underline{a}_1^{\,T} \ldots \underline{a}_n^{\,T})^T$.

Let A and B be $n \times m$ and $r \times s$ matrices having typical elements A_{jk}, B_{jk}. Define the Kronecker product $C = A \otimes B$ as the $nr \times ms$ block matrix whose (i, j)th block is $A_{ij}B$, $i = 1, \ldots, n$, $j = 1, \ldots, m$.

Then

$$\lim_{T\to\infty} f_T(\omega) = f(\omega)$$

$$\lim_{T\to\infty} \mathrm{Cov}(\mathrm{vec}(f_T(\omega_j)),\ \mathrm{vec}(f_T(\omega_k)))$$

$$= \delta_{j-k} f^T(\omega_j) \otimes f^T(\omega_k) ,$$

where $\omega_j = \frac{2\pi j}{T}$, $j = 1, \ldots, [\frac{T-1}{2}] \equiv N$, and $[x]$ denotes the greatest integer less than or equal to x.

Thus $f_T(\cdot)$ is asymptotically unbiased and $f_T(\omega_1), \ldots, f_T(\omega_N)$ are asymptotically uncorrelated. However, the precision of the estimator does not improve as $T \to \infty$ (In fact, it is independent of sample size.).

Kernel Method

Because of the inconsistency of $f_T(\cdot)$, one is led to forming an estimator by smoothing the periodogram; i.e., estimate $f(\cdot)$ be a weighted average of $f_T(\omega)$ in the neighborhood of ω. The general form of these averages is ([3])

$$f_{T,M}(\omega) = \int_{-\pi}^{\pi} K_M(\omega_0) f_T(\omega - \omega_0) d\omega_0$$

where $K_M(\cdot)$ is some weighting function called an amplitude window generator. Ease of computation is afforded by noting that

$$f_{T,M}(\omega) = \frac{1}{2\pi} \sum_{|v|\le M} k(\tfrac{v}{M}) R_T(v) e^{-iv\omega}$$

where

$$K_M(\omega) = \frac{1}{2\pi} \sum_{|v|\le M} k(\tfrac{v}{M}) e^{-iv\omega}$$

and $k(\cdot)$ is a symmetric weighting function for the $R_T(\cdot)$ called a coefficient window generator. The function $k(\cdot)$ is defined on the interval $[-1, 1]$ and is one at zero. The integer M is called the truncation point.

Among the considerations in the choice of $k(\cdot)$ and M are

1) $f_{T,M}(\omega)$ is an estimator $\hat{J}(\omega)$ of the integrated average

$$J(\omega) = \int_{-\pi}^{\pi} K_M(\omega_0) f(\omega - \omega_0) d\omega_0$$

which is approximating $f(\omega)$. Thus there are two possible sources of error in using $f_{T,M}(\omega)$: Either $\hat{J}(\omega)$ is a poor estimator of $J(\omega)$ and/or $J(\omega)$ is a poor approximation to $f(\omega)$. Intuitively, the better the estimator $\hat{J}(\omega)$ *is of* $J(\omega)$, *the less representative* $J(\omega)$ *is of* $f(\omega)$. Thus there is a trade off between variance and bias in choosing K_M and M.

2) Since $f(\cdot)$ is positive definite we would like to choose a K_M which leads to a positive definite estimator of $f(\cdot)$.

A kernel which takes these considerations into account has been suggested by Parzen [9]

$$k(x) = \begin{cases} 1 - 6x^2 + 6|x|^3 & , \quad |x| \le .5 \\ 2(1 - |x|)^3 & , \quad .5 \le |x| \le 1 \\ 0 & , \quad |x| > 1 \end{cases}$$

For truncation point M, the choice of this coefficient window generator leads to

$$K_M(\omega) = \frac{3}{4M^3} \left[\frac{\sin \frac{1}{4} M\omega}{\frac{1}{2} \sin \frac{1}{2}\omega}\right]^4 [1 - \tfrac{2}{3}(\sin \tfrac{\omega}{2})^2] .$$

Thus the kernel method leads to consistent estimators but suffers from the lack of an easily implementable method for objectively choosing M.

To use $f_{T,M}$ for tests of hypotheses and determining confidence intervals, we have

<u>Theorem</u> ([12])

$\nu f_{T,M}$ is approximately complex Wishart with dimension d, degrees of freedom

$$\nu = \frac{T}{M} \frac{1}{\int_{-1}^{1} k(u)du} ,$$

and covariance matrix $f(\omega)$.

3. Autoregressive Spectral Estimator

The problem of choosing M can be alleviated if we make the following further assumption about $f(\cdot)$.

<u>Theorem</u> (see [6])

If these exist positive constants λ_1, λ_2 such that $f(\omega) - \lambda_1 I$ and $\lambda_2 I - f(\omega)$ are positive definite for all ω, then there exist $(d \times d)$ matrices $A_\infty(0) \equiv I, A_\infty(1), A_\infty(2), \ldots,$ and Σ_∞ such that $\underline{Y}$ can be written as the infinite order multiple autoregression

$$\sum_{j=0}^{\infty} A_\infty(j)\underline{Y}(t-j) = \underline{\varepsilon}(t) , \qquad t \in Z$$

where $E(\underline{\varepsilon}(t)) = \underline{0}$ and $E(\underline{\varepsilon}(t)\underline{\varepsilon}^T(t+v)) = \delta_v \Sigma_\infty$.

Thus the $A_\infty(\cdot)$ and $R(\cdot)$ are related by the Yule-Walker equations (see [4], p. 19)

$$\sum_{j=0}^{\infty} A_\infty(j)R(j-v) = \delta_v \Sigma_\infty , \qquad v \geq 0$$

and $f(\omega)$ can be written

$$f(\omega) = \frac{1}{2\pi} G_\infty^{-1}(e^{i\omega}) \Sigma_\infty G_\infty^{-*}(e^{i\omega})$$

where

$$G_\infty(z) = \sum_{j=0}^{\infty} A_\infty(j) z^j .$$

Thus one approximates f by the p^{th} order autoregressive approximating spectral density f_p

$$f_p(\omega) = \frac{1}{2\pi} G_p^{-1}(e^{i\omega}) \Sigma_p G_p^{-*}(e^{i\omega}) ,$$

where

$$G_p(z) = \sum_{j=0}^{p} A_p(j) z^j$$

and the $A_p(\cdot)$ and Σ_p are solution of the p^{th} order Yule-Walker equations

$$\sum_{j=0}^{p} A_p(j)R(j-v) = \delta_{v,0} \Sigma_p , \qquad v \geq 0 .$$

The autoregressive estimator of f is thus found by:

1. For order p let

$$\hat{f}_p(\omega) = \frac{1}{2\pi} \hat{G}_p^{-1}(e^{i\omega}) \hat{\Sigma}_p \hat{G}_p^{-*}(e^{i\omega}) ,$$

where

$$\hat{G}_p(z) = \sum_{j=0}^{p} \hat{A}_p(j) z^j$$

and the $\hat{A}_p(\cdot)$ and $\hat{\Sigma}_p$ are obtained by solving the p^{th} order Yule-Walker equations with the sample autocovariances $R_T(v)$ replacing the $R(\cdot)$.

2. To choose the "best" order $\hat{p}$, Parzen [13] suggests $\hat{p}$ which minimizes

$$CAT(p) = tr[\frac{d}{T} \sum_{j=1}^{p} (\frac{T-jd}{T}) \hat{\Sigma}_j^{-1} - \frac{T-pd}{T} \hat{\Sigma}_p^{-1}] ,$$

for p = 1, ..., maximum order M.

Again there are two types of error possible: (1) $\hat{G}_{\hat{p}} - G_p$ and (2) $G_p - G_\infty$. The function

$$J(p) = tr[\frac{d}{T} \sum_{j=1}^{p} \Sigma_j^{-1} - \Sigma_p^{-1}]$$

which CAT(p) is estimating is a measure of the mean square error of using $\hat{G}_{\hat{p}}$ to estimate the approximating transfer function G_∞.

Akaike [1], approaching the order determination problem as estimating a true autoregressive order p, suggests using the value $\hat{p}$ minimizing

$$AIC(p) = \log|\hat{\Sigma}_p| + 2d^2p/T ,$$

where $|A|$ denotes the determinant of A. For a comparison of the CAT and AIC criteria, see [13].

4. Periodic Autoregressive Spectral Estimator

For a p^{th} order approximating autoregressive process there are $pd^2 + \frac{d(d+1)}{2}$ scalar parameters to be estimated; a number that can be prohibitively large for small samples. Further there are many situations where a large proportion of these parameters are in fact negligible in size. One can capitalize on these considerations by noting (see [7]) that a p^{th} order multiple autoregression with parameters Σ_p, $A_p(1), \ldots, A_p(p)$ can be written as the univariate periodically stationary autoregression

$$\sum_{j=0}^{p_t} \alpha_t(j)X(t-j) = \eta(t) , \qquad t\epsilon Z$$

where

1. $X(t) = Y_\ell(r)$, $\ell = mod(t-1, d) + 1$, $r = [\frac{t}{d}] + 1$,

where mod(j, k) denotes the remainder when j is divided by k,

2. $E(\eta(t)) = 0$, $E(\eta(t)\eta(t+v)) = \delta_v\sigma_t^2$

3. $p_t = p_t + kd$, $\alpha_t(j) = \alpha_{t+kd}(j)$, $\sigma_t^2 = \sigma_{t+kd}^2$

4. $p_t = dp + mod(t-1, d)$

5. $\sigma_k^2 = D_{kk}$

$$\alpha_k(j) = \begin{cases} L_{k,k-j} & j < k = 1, \ldots, d \\ A'_{k,d-mod(j-k,d)}([\frac{j-k}{d}] + 1) , & j \geq k+1, \ldots, p_k \end{cases}$$

where

$$\Sigma_p = L^{-1} DL^{-T}$$

L is unit lower triangular, $D = diag(D_{11}, \ldots, D_{dd})$ and $A'_p(v) = LA_p(v)$, $v = 1, \ldots, p$.

This process X is similar to a scalar autoregression except the order, coefficients, and residual variances are the same for like components in $\underset{\sim}{Y}$ and different for different components.

The periodically stationary autoregressive method estimates the parameters of the X series and then transforms these back to the multiple parameters according to the inverse transformation

$$A(j) = L^{-1}A'(j)\ , \qquad j = 1, \ldots, p$$

$$L_{kj} = \alpha_k(k - j)\ , \qquad j \leq k = 1, \ldots, d$$

$$A'_{kj}(v) = \alpha_k(dv - j + k)\ , \qquad v = 1, \ldots, p;\ j, k = 1, \ldots, d$$

Also, removing the restriction that $p_t = dp + \mathrm{mod}(t - 1, d)$ allows the introduction of zeros into the $A_p(\cdot)$.

To estimate the parameters of the periodic autoregression we note that they satisfy a Yule-Walker type equation with the $R(\ell, m) = \mathrm{Cov}(X(\ell), X(m))$:

$$\sum_{j=0}^{p_k} \alpha_k(j)R(k - j, k - v) = \delta_{v,0}\, \sigma_k^{\,2}\ , \quad v \geq 0\ .$$

Then for order p for component k, estimators $\hat{\alpha}_{k,p}(1), \ldots, \hat{\alpha}_{k,p}(p)$, and $\hat{\sigma}^2_{k,p}$ are obtained by solving the sample analog of the Yule-Walker type equations

$$\sum_{j=0}^{p} \hat{\alpha}_{k,p}(j)R_{Td}(k - j, k - v) = \delta_{v,0}\, \hat{\sigma}^2_{k,p}\ , \quad v = 0, \ldots, p$$

where

$$R_{Td}(k, v) = \frac{1}{T} \sum_{j=0}^{T-[\frac{k+v}{d}]} X(k + dj)X(v + dj)\ ,$$

$k = 1, \ldots, d,\ v = 0, \ldots, Td - k + 1$.

By inspection it is clear that the $R_{Td}(\cdot, \cdot)$ can be obtained from the multiple sample autocovariances $R_T(\cdot)$ by

$$R_{Td}(k, v) = R_{T,\ell m}(s - r)\ ,$$

where $\ell = \mathrm{mod}(k - 1, d) + 1$, $m = \mathrm{mod}(v - 1, d) + 1$, $r = [\frac{k}{d}] + 1$, and $s = [\frac{v}{d}] + 1$.

For a given component k, the best order p is chosen to minimize the PCAT criterion

$$\mathrm{PCAT}(k, p) = \sum_{j=0}^{k-1} \left[\frac{1}{T} \sum_{\ell=1}^{p} \frac{\hat{\alpha}^2_{k,\ell}(j)}{\hat{\sigma}^2_{k,\ell}} - \frac{\hat{\sigma}^2_{k,p}(j)}{\hat{\sigma}^2_{k,p}} \right]\ , \quad p = 1, \ldots, M\ .$$

Once the univariate estimators have been obtained and transformed back to the parameters $\hat{\Sigma}_{\hat{p}}$, $\hat{A}_{\hat{p}}(1), \ldots, \hat{A}_{\hat{p}}(\hat{p})$ of the multiple autoregression (with $\hat{p} = [\max(\hat{p}_j - j)/d] + 1$), the multiple parameters are used to find the estimator of f.

5. A Worked Example

To illustrate the methodology presented above, we consider monthly total ozone levels in the atmosphere above four cities (see [5]): Arosa, Switzerland; Aspendale, Australia; Huancayo, Peru; and Kodaikanal, India.

Figure 1 plots the four individual series. The plot indicates the series have periodic behavior and that the Arosa and Aspendale series are more variable than the Huancayo and Kodaikanal series. Table 1 presents the monthly means and standard deviations for the four series together with their overall means and standard deviations. Inspection of the Arosa and Aspendale columns confirms that the series behave cyclically with levels peaking at Arosa in April and at Aspendale in September. Though less variable than Arosa and Aspendale, Huancayo and Kodaikanal exhibit some periodic behavior.

The multiple time series methods are illustrated by analyzing two four dimensional series: Series 1 is the common stretch of the overall mean adjusted series, while Series 2 is the common stretch of the monthly

mean adjusted series. Arosa, Aspendale, Huancayo, and Kodaikanal occupy the first through fourth components respectively.

Table 2 summarizes the auto and cross correlation functions $\rho_{jk}(v) = R_{jk}(v)/(R_{jj}(0)R_{kk}(0))^{\frac{1}{2}}$ for the two multiple series. All four autocorrelation functions for lags 0, ..., 24 are given, while the cross correlations are illustrated by presenting those for Arosa-Aspendale and Arosa-Kodaikanal. The autocorrelations for the overall mean adjusted series exhibit the expected cyclic pattern, and the monthly mean adjustment eliminates this pattern.

The cross correlations for the overall mean adjusted Arosa and Aspendale series have peaks at $v = 5$ and $v = -7$, emphasizing how the two series are nearly identical but out of phase. The cross correlations for the monthly mean adjusted series indicate that little if any cross correlation remains among the univariate series.

A summary of the multiple spectral estimation is summarized in figures 2 through 9 and tables 3 and 4. The Parzen window estimator with truncation point 24 as well as the two types of autoregressive methods are used. The graphs for the three methods are labelled PARZ, CAT, and PCAT.

Table 3 presents the CAT criterion determined order autoregressive models fit to the two multiple series. Table 4 gives the corresponding PCAT criterion determined order periodically stationary autoregressive models. Notice that 122 and 26 scalar parameters are required for the autoregressive models while 37 and [illegible] are required for the corresponding periodically stationary autoregressive models. The estimates of Σ are also standardized so that the estimators for the mean adjusted series can be compared as can the estimators for the monthly mean adjusted series.

Figures 2 through 9 represent:

Figures 2 and 3: Power spectra estimates for mean detrended Arosa and Kodaikanal series.

Figures 4 and 5: Power spectra estimates for monthly mean detrended Arosa and Kodaikanal series.

Figures 6 and 7: Estimates of squared coherency between mean detrended Arosa and Aspendale and Arosa and Kodaikanal series.

Figures 8 and 9: Estimates of squared coherency between monthly mean detrended Arosa and Aspendale and Arosa and Kodaikanal series.

The power spectra estimates are plotted on a logarithmic scale and are standardized to integrate to 1.

Inspection of these figures leads to the following qualitative conclusions:

1) The stationary autoregressive estimator appears to perform well; with sharp peaks at expected frequencies.
2) The periodically stationary autoregressive estimator appears to be a smoother version of the stationary autoregressive estimator. This is not unexpected since far fewer parameters are used. However, the basic shape of the spectra is exhibited and the method appears to be useful, particularly in series having relatively short length, or in series where some components are close to white noise and others require long labs.
3) Removal of monthly means leads to basically flat spectra and strong evidence of noncoherence between the series.

6. A Computer Program for Multiple Spectral Estimation

MULTSP is a main program written in Fortran to analyze multiple time series in both the time domain and the frequency domain. The program uses the multiple time series subprograms in the TIMESBOARD subroutine library written by the author (see [14]) for a description of the univariate time series subroutine library).

MULTSP consist of the following parts:

1) Detrending (according to a variety of methods) of univariate series and formation of multiple series.
2) Calculation (via a Fast Fourier Transform Algorithm) and display of multiple autocorrelations.
3) Calculation and display of windowed spectral estimators.
4) Calculation and display of autoregressive model and spectral estimators.
5) Calculation and display of periodically stationary autoregressive models and spectral estimators.

MULTSP generates both printer plots and plots (such as figures 1 through 9) on a CALCOMP plotter.

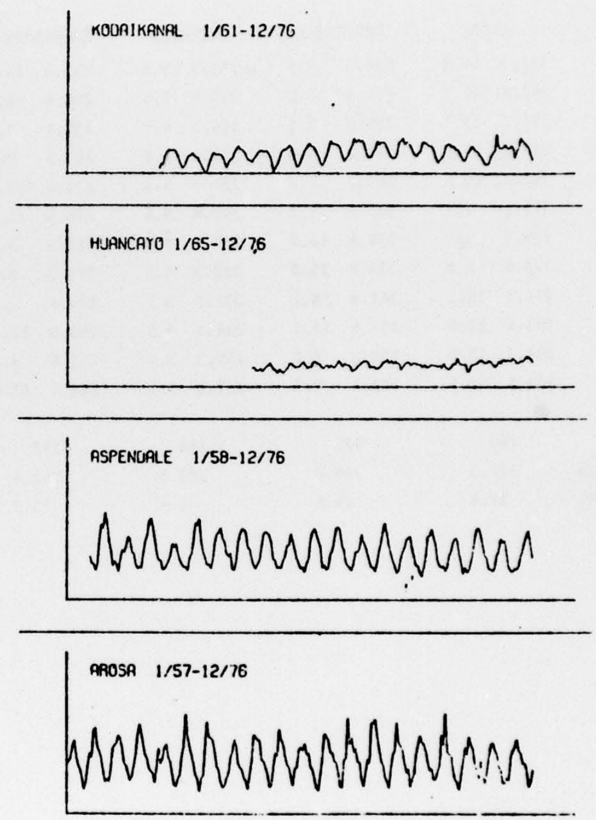

Figure 1. Four Monthly Total Ozone Series.

Table 1. Means and Standard Deviations for Ozone Data

	Arosa		Aspendale		Huancayo		Kodaikanal	
Jan	337.3	14.8	296.6	9.5	263.9	5.5	243.3	13.0
Feb	363.8	27.2	291.6	8.1	265.0	7.4	248.4	11.2
Mar	374.2	25.3	286.0	5.1	264.3	6.9	255.4	8.5
Apr	379.1	17.1	282.2	4.6	259.6	6.0	267.7	9.1
May	361.2	10.6	299.7	8.7	259.7	5.4	274.6	10.2
Jun	345.1	7.9	320.7	12.2	260.8	5.6	278.1	11.3
Jul	325.2	6.8	338.8	16.9	263.9	5.0	273.6	9.2
Aug	313.8	7.3	353.9	15.5	268.3	4.1	273.3	9.1
Sep	296.3	10.1	361.8	16.1	271.3	5.1	270.8	9.3
Oct	283.6	13.6	354.6	13.2	271.3	5.3	265.0	10.3
Nov	287.3	12.0	330.1	8.8	268.2	5.4	253.8	9.8
Dec	308.5	12.7	312.4	9.8	267.0	5.1	244.4	12.0
N	240		228		144		192	
Overall Mean	331.3		319.0		265.3		262.4	
Overall S.D.	35.8		29.6		6.9		15.9	

Table 2. Auto and Cross Correlations

Series 1: Arosa, Series 2: Aspendale,

Series 3: Huancayo, Series 4: Kodaikanal

Mean Adjusted:

v	$\rho_{11}(v)$	$\rho_{22}(v)$	$\rho_{33}(v)$	$\rho_{44}(v)$	$\rho_{12}(v)$	$\rho_{12}(-v)$	$\rho_{14}(v)$	$\rho_{14}(-v)$
0	1.00	1.00	1.00	1.00	-.72	-.72	.03	.03
1	.71	.82	.74	.81	-.43	-.83	.41	-.33
2	.43	.48	.52	.53	-.03	-.69	.64	-.57
3	-.02	.03	.34	.19	.36	-.39	.70	-.63
4	-.37	-.39	.22	-.08	.68	.02	.54	-.55
5	-.63	-.70	.16	-.28	.81	.44	.30	-.30
6	-.71	-.82	.12	-.35	.75	.72	-.02	-.02
7	-.67	-.71	.13	-.28	.46	.80	-.31	.32
8	-.40	-.43	.15	-.11	.06	.67	-.52	.55
9	-.06	-.03	.25	.11	-.36	.37	-.65	.68
10	.37	.37	.34	.40	-.65	-.02	-.59	.64
11	.63	.68	.44	.62	-.78	-.40	-.34	.39
12	.79	.80	.46	.72	-.68	-.65	.01	.06
13	.67	.69	.40	.61	-.43	-.75	.37	-.30
14	.38	.39	.28	.36	-.08	-.63	.60	-.54
15	.02	-.01	.13	.06	.29	-.35	.64	-.60
16	-.34	-.40	.03	-.21	.60	.05	.52	-.53
17	-.59	-.67	-.01	-.39	.73	.42	.27	-.31
18	-.66	-.78	-.07	-.45	.69	.69	-.01	-.03
19	-.60	-.67	-.08	-.37	.42	.76	-.30	.26
20	-.38	-.39	-.08	-.20	.04	.62	-.48	.51
21	-.05	-.01	.00	.04	-.33	.35	-.58	.62
22	.31	.37	.05	.31	-.62	-.02	-.51	.61
23	.60	.67	.17	.53	-.71	-.34	-.29	.39
24	.69	.79	.22	.62	-.64	-.60	.01	.07

Monthly Mean Adjusted:

v	$\rho_{11}(v)$	$\rho_{22}(v)$	$\rho_{33}(v)$	$\rho_{44}(v)$	$\rho_{12}(v)$	$\rho_{12}(-v)$	$\rho_{14}(v)$	$\rho_{14}(-v)$
0	1.00	1.00	1.00	1.00	.01	.01	.05	.05
1	.05	.47	.73	.79	.03	.09	.10	.08
2	.09	.40	.61	.70	.15	.03	.04	.11
3	.00	.30	.55	.60	.10	.04	.11	.17
4	.12	.31	.55	.58	.12	.10	.01	.13
5	.12	.27	.50	.55	.02	-.02	.04	.24
6	.15	.23	.46	.54	-.01	-.09	.00	.16
7	-.07	.20	.43	.52	-.10	-.04	.13	.29
8	.05	.18	.40	.49	.01	.01	.20	.16
9	-.08	.16	.40	.40	-.01	.03	.04	.20
10	.14	-.01	.36	.41	.02	-.02	-.04	.23

Table 2 (Continued)

v	$\rho_{11}(v)$	$\rho_{22}(v)$	$\rho_{33}(v)$	$\rho_{44}(v)$	$\rho_{12}(v)$	$\rho_{12}(-v)$	$\rho_{14}(v)$	$\rho_{14}(-v)$
11	-.09	-.02	.32	.37	-.09	-.08	.02	.17
12	.10	.00	.25	.37	.03	.05	.03	.17
13	.00	.06	.26	.31	-.08	.03	.15	.14
14	-.02	-.02	.25	.24	-.11	.08	.09	.15
15	.09	-.01	.21	.21	-.13	.03	.10	.17
16	.04	-.04	.20	.20	-.06	.07	.08	.09
17	-.07	.03	.20	.20	.03	.05	.03	.15
18	.08	.02	.14	.22	.08	.10	.06	.14
19	.02	.10	.12	.25	-.10	.23	.04	.08
20	-.03	.12	.08	.25	-.10	.12	.14	.06
21	.02	.18	.06	.23	.01	.13	.05	.02
22	.05	.18	-.03	.23	-.07	.03	-.02	.09
23	.02	.18	-.01	.26	.04	.15	.05	.07
24	-.10	.16	-.05	.25	.00	.11	-.01	.08

Table 3. Fitted Autoregressive Models

Mean Adjusted: $\hat{p} = 7$

$$\hat{\Sigma}_7 = \begin{bmatrix} .16 & & & \\ -.02 & .09 & & \\ .02 & .02 & .31 & \\ .00 & .02 & .04 & .14 \end{bmatrix} \qquad \hat{A}_7(1) = \begin{bmatrix} -.16 & .24 & .03 & .22 \\ -.03 & -.66 & .04 & -.01 \\ .10 & -.05 & -.60 & -.10 \\ -.08 & .32 & -.04 & -.65 \end{bmatrix}$$

$$\hat{A}_7(2) = \begin{bmatrix} -.30 & -.09 & .17 & -.03 \\ -.05 & -.11 & .00 & -.04 \\ .33 & .16 & .00 & .01 \\ -.08 & -.03 & .04 & .01 \end{bmatrix} \qquad \hat{A}_7(3) = \begin{bmatrix} .11 & .17 & -.10 & -.03 \\ -.02 & .10 & .04 & .13 \\ -.21 & -.05 & .10 & .14 \\ -.10 & -.08 & .00 & .11 \end{bmatrix}$$

$$\hat{A}_7(4) = \begin{bmatrix} .12 & .14 & -.08 & .07 \\ -.11 & .04 & -.02 & -.20 \\ .03 & .21 & -.12 & -.14 \\ .11 & .25 & -.04 & -.11 \end{bmatrix} \qquad \hat{A}_7(5) = \begin{bmatrix} -.13 & -.06 & .01 & -.29 \\ -.06 & -.02 & .02 & .05 \\ .07 & .01 & -.05 & -.09 \\ -.12 & .11 & -.04 & -.02 \end{bmatrix}$$

$$\hat{A}_7(6) = \begin{bmatrix} -.12 & -.23 & .00 & .11 \\ -.09 & .05 & -.14 & .19 \\ -.30 & -.18 & -.04 & .11 \\ -.11 & -.03 & .11 & -.07 \end{bmatrix} \qquad \hat{A}_7(7) \begin{bmatrix} .22 & -.11 & .11 & -.18 \\ .03 & .07 & -.01 & .09 \\ .08 & .15 & -.10 & .00 \\ -.11 & -.22 & -.16 & .04 \end{bmatrix}$$

Monthly Mean Adjusted: $\hat{p} = 1$

$$\hat{\Sigma}_1 = \begin{bmatrix} .93 & & & \\ -.02 & .75 & & \\ .06 & .01 & .46 & \\ .01 & -.06 & .08 & .34 \end{bmatrix} \qquad \hat{A}_1(1) = \begin{bmatrix} -.02 & -.13 & .22 & -.22 \\ -.03 & -.38 & .06 & .18 \\ .04 & .08 & -.68 & -.06 \\ -.07 & .16 & -.04 & -.71 \end{bmatrix}$$

Table 4. Fitted Periodically Stationary Autoregressive Models

Mean Adjusted: $\hat{p}_1 = 4$, $\hat{p}_2 = 12$, $\hat{p}_3 = 10$, $\hat{p}_4 = 7 \Rightarrow \hat{p} = 3$

$$\hat{\Sigma}_3 = \begin{bmatrix} .25 & & & \\ -.04 & .14 & & \\ .02 & .05 & .36 & \\ .00 & .04 & .05 & .19 \end{bmatrix} \qquad \hat{A}_3(1) = \begin{bmatrix} -.30 & .54 & .05 & .21 \\ .03 & -.90 & .04 & -.01 \\ .01 & -.29 & -.59 & -.15 \\ -.26 & .12 & -.04 & -.76 \end{bmatrix}$$

$$\hat{A}_3(2) = \begin{bmatrix} .00 & .00 & .00 & .00 \\ -.14 & -.09 & .05 & -.06 \\ .33 & .24 & -.05 & .00 \\ .01 & .01 & .01 & -.01 \end{bmatrix} \qquad \hat{A}_3(3) = \begin{bmatrix} .00 & .00 & .00 & .00 \\ .00 & .34 & .01 & -.06 \\ .00 & .15 & .00 & -.03 \\ .00 & .10 & .00 & -.02 \end{bmatrix}$$

Monthly Mean Adjusted: $\hat{p}_1 = 0$, $\hat{p}_2 = 4$, $\hat{p}_3 = 4$, $\hat{p}_4 = 5 \Rightarrow \hat{p} = 1$

$$\hat{\Sigma}_1 = \begin{bmatrix} 1.00 & & & \\ -.02 & .75 & & \\ .05 & -.02 & .47 & \\ .00 & -.09 & .09 & .37 \end{bmatrix} \qquad \hat{A}_1(1) = \begin{bmatrix} .00 & .00 & .00 & .00 \\ .00 & -.39 & .06 & .17 \\ .00 & .01 & -.70 & -.07 \\ .00 & .05 & -.03 & -.76 \end{bmatrix}$$

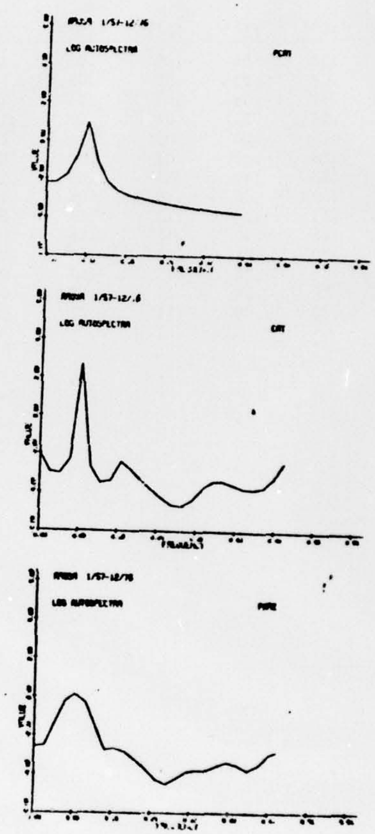

Figure 2. Log Power Spectra Estimates for Mean Adjusted Arosa Series

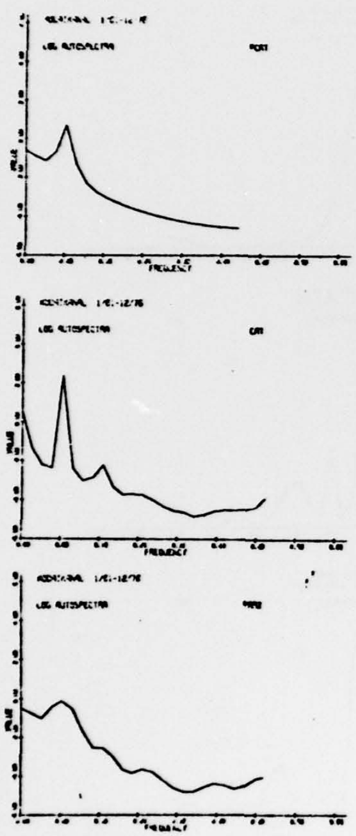

Figure 3. Log Power Spectra Estimates for Mean Adjusted Kodaikanal Series

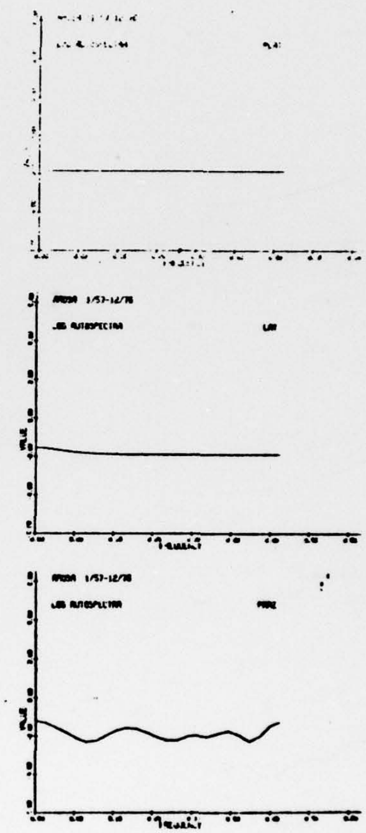

Figure 4. Log Power Spectra Estimates for Monthly Mean Adjusted Arosa Series

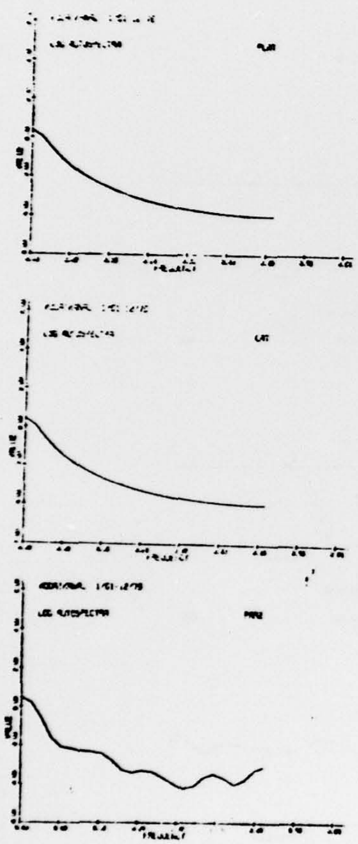

Figure 5. Log Power Spectra Estimates for Monthly Mean Adjusted Kodaikanal Series

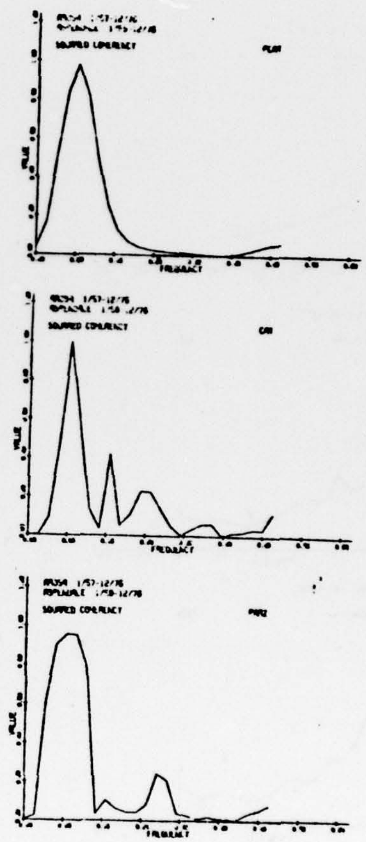

Figure 6. Squared Coherency Estimates for Mean Adjusted Arosa, Aspendale Series

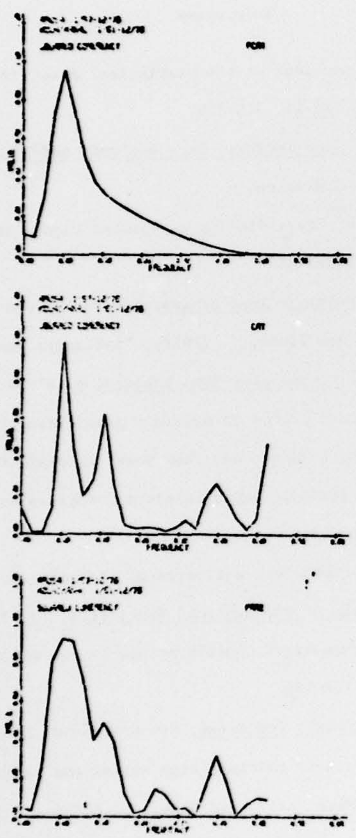

Figure 7. Squared Coherency Estimates for Mean Adjusted Arosa, Kodaikanal Series

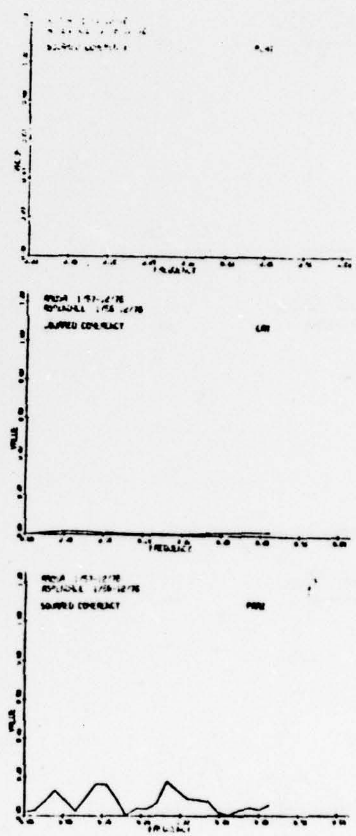

Figure 8. Squared Coherency Estimates for Monthly Mean Adjusted Arosa, Aspendale Series (PCAT estimate identically zero)

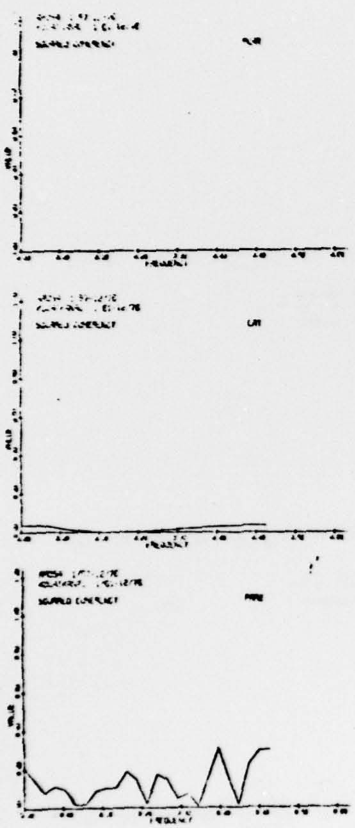

Figure 9. Squared Coherency Estimates for Monthly Mean Adjusted Arosa, Kodaikanal Series (PCAT estimate identically zero)

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